Quiz 3 — MODEL SOLUTIONS

1. Prove, using a truth table, that $A \vee (B \wedge C)$ is logically equivalent to $(A \vee B) \wedge (A \vee C)$.

A	B	C	$B \wedge C$	$A \vee (B \wedge C)$	$A \vee B$	$A \vee C$	$(A \vee B) \wedge (B \vee C)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The fact that the columns labeled $A \vee (B \wedge C)$ and $(A \vee B) \wedge (B \vee C)$ are identical establishes that the two expressions are logically equivalent.

2. Susan is having trouble remembering her brother's phone number. She knows that the area code is 401 and she is sure that in the last four digits at least one digit is repeated. If she has to try all possible such phone numbers before successfully reaching her brother, how many phone numbers will she need to dial?

Solution: There are $10 \times 10 \times 10$ ways to choose the first three digits (area code is already set.) We want to multiply this by the number of four digit numbers that include at least one repeated digit. There are 10^4 possible four digit numbers in all. Of these, $(10)_4 = 10 \times 9 \times 8 \times 7$ contain *no* repeated digit. Therefore the difference of these two numbers, i.e.

$$10^4 - (10)_4$$

gives the desired number. The answer is therefore $10 \times 10 \times 10 \times (10^4 - (10_4))$.