Quiz 7 — MODEL SOLUTIONS

1. Use mathematical induction to prove that the following formula holds for all integers $n \in \mathbb{Z}^+$:

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n \cdot (3n-1)}{2}$$

We will use induction to prove that the formula holds for all $n \geq 1$.

Basis step: When $n = 1$, the sum on the left goes from 1 to $3 \cdot 1 - 2 = 1$, in other words it is simply the first term, and so equals 1. The right hand side is $\frac{1 \cdot (3-1)}{2} = \frac{1 \cdot 2}{2} = 1$, as required.

Induction hypothesis: Assume the formula holds for $n = k$:

$$1 + 4 + 7 + \cdots + (3k-2) = \frac{k(3k-1)}{2}$$

We want to show the formula holds for $n = k + 1$:

$$1 + 4 + 7 + \cdots + (3[k+1]-2) = \frac{[k+1](3[k+1]-1)}{2} = \frac{(k+1)(3k+2)}{2}$$

Observe that $1 + 4 + 7 + \cdots + (3[k+1]-2)$

$$\begin{aligned}
&= 1 + 4 + 7 + \cdots + (3k-2) + (3[k+1]-2) \\
&= [1 + 4 + 7 + \cdots + (3k-2)] + (3k+1) \\
&= \frac{k(3k-1)}{2} + (3k+1), \text{ by the induction hypothesis} \\
&= \frac{(3k^2-k) + (6k+2)}{2} \\
&= \frac{3k^2+5k+2}{2} \\
&= \frac{(k+1)(3k+2)}{2}
\end{aligned}$$

Therefore by the **principle of mathematical induction**, the formula holds for all $n \geq 1$.