Name ________________ Seat ________

QUIZ 8 MthSc 119 sec.1

April 8, 2005

1. Define the sequence $e_0, e_1, e_2, \ldots$ as follows:

$$e_0 = 1,\ e_1 = 4 \text{ and } e_n = 4(e_{n-1} - e_{n-2}) \text{ for } n \geq 2.$$

a What are the first five terms of the sequence (i.e., what are e_0, e_1, e_2, e_3 and e_4)?

$e_0 = 1$, $e_1 = 4$, $e_2 = 12$, $e_3 = 32$ and $e_4 = 80$.

b Prove using mathematical induction (strong version):
$e_n = (n+1)2^n$ for all integers $n \geq 0$.

We will use strong induction to show that $e_n = (n+1)2^n$ for all $n \in \mathbb{N}$.
Basis cases: When $n = 0$, $e_0 = 1$ and $(0+1)2^0 = 1 \times 1 = 1$, as required. When $n = 1$, $e_1 = 4$ and $(1+1)2^1 = 2 \times 2 = 4$, as required.
Strong Induction hypothesis: Suppose that the formula $e_n = (n+1)2^n$ holds for all values of n from 0 to k. We need to prove that $e_{k+1} = [(k+1)+1]2^{k+1} = (k+2)2^{k+1}$. Observe that

$$\begin{aligned} e_{k+1} &= 4(e_k - e_{k-1}), \text{ by definition} \\ &= 4[\,(k+1)2^k - k2^{k-1}\,], \text{ by the induction hypothesis} \\ &= 2(k+1)2^{k+1} - k2^{k+1} \\ &= [2(k+1) - k]2^{k+1} \\ &= (k+2)2^{k+1} \end{aligned}$$

as required. Therefore by the **principle of mathematical induction, strong version**, the inequality holds for all $n \in \mathbb{N}$. QED