Spring 2005 MthSc 119, Section 1 Test #1 (50 points total)

Name Mr. Key Seat ________________

This exam is closed book, closed notes. You may not store formulas on your calculator. Answer in the space provided. *Show your work* (in logical order and using correct notation) for full credit. Indicate your final answer clearly.

1. (4 points) Use a truth table to determine whether or not the Boolean expression $\neg(\neg A \wedge B)$ is logically equivalent to $B \to A$.

A	B	$\neg A$	$\neg A \wedge B$	$\neg(\neg A \wedge B)$	$B \to A$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	F	T	T

The two statements are equivalent because the corresponding columns in the truth table are identical.

2. (4 points) Show that the sum of an even number and an odd number is odd.

We will show that the sum of an even number and an odd number is odd. Let x be an even number and let y be an odd number. By definition of even, $2 | x$. By definition of divisible $x = 2a$ for some integer a. By definition of odd $y = 2b+1$ for some integer b. Observe that $x + y = 2a + (2b+1) = 2(a+b)+1$. Hence there is a number c, namely $a+b$, so that $x+y = 2c+1$. Therefore by the definition of odd, $x+y$ is odd. ☺

3. (2 points) State the hypothesis and the conclusion for the statement:

A figure is a square only if it is a rectangle.

hypothesis: A figure is a square.

conclusion: A figure is a rectangle.

4. (6 points) Let x be an integer. Show that x is odd if and only if $x + 1$ is even.

We will show that for any integer x, x is odd if and only if $x+1$ is even. Let x be an integer.

($\Rightarrow$) Assume x is odd. By definition of odd, there is a number b so that $x = 2b+1$. Observe that $x+1 = (2b+1)+1 = 2(b+1)$. Hence there is an integer c, namely $b+1$, so that $x+1 = 2c$. Therefore $x+1$ is even by definition of even.

($\Leftarrow$) Assume $x+1$ is even. So $2 \mid (x+1)$. By def. of divisible there is an integer a with $x+1 = 2a$. Observe that $x = 2a-1 = 2a-2+1 = 2(a-1)+1$. Hence there is an integer d, namely $a-1$ so that $x = 2d+1$, so x is odd by def. of odd.

5. (3 points) Find the number of positive divisors of $10140 = 2^2 \times 3^5 \times 13^2$.

The number of divisors is $(2+1)(5+1)(2+1)$
$= 3 \times 6 \times 3 = \boxed{54}$.

6. (4 points) Prove or disprove: If a, b, and c are positive integers with $a|(bc)$ then $a|b$ or $a|c$.

Not true.

Counter-example: Let $a = 4$, $b = 2$, $c = 2$.
$4 \mid 2 \cdot 2$, but $4 \nmid 2$ and $4 \nmid 2$.

7. (3 points) Let $A = \{c, d, e, f\}$ and let $B = \{a, b, c, d, e\}$. Compute:

(a) $A \cup B = \{a, b, c, d, e, f\}$

(b) $A \cap B = \{c, d, e\}$

(c) $|2^B| = 2^{|B|} = 2^5 = 32$

8. (4 points) Write using the quantifier $\exists$ and/or $\forall$:

   (a) There is an integer that when multiplied by any integer always gives the result 0.

   $\exists x, \forall y; x \cdot y = 0.$

   (b) No matter what integer you choose, there is always another integer that is larger.

   $\forall x, \exists y, y > x.$

9. (4 points) Find the cardinality of each of the following sets:

   (a) $|\{x \in \mathbb{Z} : x \in \emptyset\}| = 0$
   (b) $|\{x \in \mathbb{Z} : \emptyset \subseteq \{x\}\}| = \infty$
   (c) $|\{x \in 2^{\{a,b,c,d\}} : |x| = 3\}| = 4$
   (d) $|\{1, 2, 2, 3\}| = 3$

10. (4 points) Let $A = \{x \in \mathbb{Z} : 12|x\}$ and $B = \{x \in \mathbb{Z} : 3|x\}$. Prove that $A \subseteq B$.

Let $A = \{x \in \mathbb{Z} : 12 | x\}$ and let $B = \{x \in \mathbb{Z} : 3 | x\}$. We will show that $A \subseteq B$. Assume $x \in A$. By definition of A, $12 | x$. By definition of divisible, $x = 12a$ for some integer a. Observe that $x = 12a = 3(4a)$. Hence there is an integer b, namely $4a$, so that $x = 3b$. So by definition of divisible, $3 | x$. Therefore $x \in B$. We conclude that $A \subseteq B$.

11. (4 points) Mark each of the following as either TRUE or FALSE:

   (a) ~~TRUE~~ FALSE $0! = 0$
   (b) ~~TRUE~~ FALSE $\{1, 2, 3\} \subseteq 2^{\{1,2,3\}}$
   (c) TRUE FALSE $\emptyset \in \{\emptyset\}$
   (d) TRUE FALSE $\{1, 2, 3, 4\} \subset \{1, 2, 3, 4, 4\}$

12. (2 points) A group of students are lining up to buy tickets for the homecoming football game. There are 6 seniors, 4 juniors, 5 sophomores and 8 freshmen.

   (a) How many different ways can the students line up? (Your answer is allowed to use the factorial symbol.)

   $$(6+4+5+8)! = 23!$$

   (b) How many ways can they line up if all the seniors must come before all the juniors who must come before all the sophomores who must come before all the freshmen? (Again, your answer is allowed to use the factorial symbol.)

   $$6! \cdot 4! \cdot 5! \cdot 8!$$

13. (4 points) A social security number consists of 9 digits each between 0 and 9. To keep things simple assume that 000 00 0000 is a legitimate social security number.

   (a) How many possible social security numbers are there?

   $$10^{10}$$

   (b) How many contain no 8s?

   $$10^9$$

   (c) How many contain at least one 8?

   $$10^{10} - 10^9$$

   (d) How many contain at least one even digit (remember that 0 is even).

   $$10^{10} - 10^5$$

14. (2 points) You own three different rings. You wear all three rings but not two of the rings on the same finger and no ring on your thumbs. Assuming any ring will fit on any finger, how many ways can you wear the rings?

   we need to make a list of length three from the set of all eight fingers (all but the thumbs.) The answer is therefore $(8)_3 = 8 \times 7 \times 6 = 336$.