PAlmetto **J**oint **A**rithmetic, **M**odularity, and **A**nalysis **S**eries **III**

SCHEDULE OF ACTIVITIES

Talks will take place on Zoom. All times are listed in EDT

Saturday, September 25, 2021

9:00–9:10am	Welcome and Introduction
9:10–10:10am	**Cecília Salgado**, University of Groningen *Mordell-Weil rank jumps on families of elliptic curves*
10:10–10:30am	Coffee Break
10:30–11:30am	**Wanlin Li**, CRM Montreal *A generalization to Elkies's theorem*
11:40–12:00pm	**Anwesh Ray**, University of British Columbia *Constructing Galois representations ramified at one prime*
12:00-1:30pm	Lunch
1:30–2:30pm	**Evan O'Dorney**, University of Notre Dame *Reflection theorems for number rings*
2:40–3:00pm	**Cuyler Warnock**, University of South Carolina *Hecke operators on modular forms with eta-multiplier*
3:00–3:20pm	Break
3:30–3:50pm	**Pavel Čoupek**, Purdue University *Ramification bounds for mod-p étale cohomology via prismatic cohomology*
4:00–4:20pm	**Santiago Arango-Piñeros**, Emory University *Mertens' theorem for Chebotarev sets*
4:30pm	**END OF DAY 1**

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Sunday, September 26, 2021

9:00–10:00am	**Matilde Lalín**, University of Montreal *Sums of certain arithmetic functions over $\mathbb{F}_q[T]$ and symplectic distributions*
10:00–10:30am	Coffee Break
10:30–10:50am	**Sudhir Pujahari**, University of Warsaw *An all-purpose Erdős-Kac theorem*
11:00–11:20am	**Amod Agashe**, Florida State University *Darmon points over arbitrary number fields*
11:30–11:50am	**Duncan Buell**, University of South Carolina 3-*Sylow subgroups of class groups of quadratic fields*
12:00-1:30pm	Lunch
1:30–2:30pm	**Ashvin A. Swaminathan**, Princeton University 2-*Selmer groups,* 2-*class groups, and the arithmetic of binary forms*
2:30–2:50pm	**Manami Roy**, Fordham University *Rational elliptic curves with non-trivial torsion*
3:00–3:20pm	**Zhining Wei**, Ohio State University *Linear relations of Siegel-Poincaré series and nonvanishing of the central value of spinor L-functions*
3:30–3:50pm	**Shaoyun Yi**, University of South Carolina *Siegel modular forms of level 4*
4:00pm	**END OF CONFERENCE**

Abstracts

Amod Agashe, Florida State University
Darmon points over arbitrary number fields

Darmon points (also called Stark-Heegner points) are certain points on elliptic curves that are defined by transcendental means, but are conjectured to be algebraic. They are generalizations or analogs of Heegner points, which played a crucial role in the resolution of Birch and Swinnerton-Dyer's conjecture for analytic rank zero or one. Darmon points were originally defined over a limited class of number fields. We shall sketch their construction over arbitrary number fields by a geometric method (assuming some conjectures); this is part of joint work with Mak Trifkovic.

Santiago Arango-Piñeros, Emory University
Mertens' theorem for Chebotarev sets

We generalize Mertens' product theorem to Chebotarev sets of prime ideals in Galois extensions of number fields. Joint work with Christoper Keyes and Daniel Keliher.

Duncan Buell, University of South Carolina
3-*Sylow subgroups of class groups of quadratic fields*

In a recent JNT paper by Kishi and Komatsu, it is shown that the quadratic fields of discriminant $4 - 3^{18n+3}$ have a class group whose 3-Sylow subgroup is of rank at least three. Kishi and Komatsu prove their result using some heavy mathematics. In our presentation, we will show that their result can "almost" (for an appropriate definition of "almost") be obtained using methods that would have been available to (and largely invented by) Gauss. We will also present a conjecture that perhaps one could hope to prove that the rank of a 3-SSG of a quadratic class group could be arbitrarily large and present parametric instances of such groups.

Pavel Čoupek, Purdue University
Ramification bounds for mod-p étale cohomology via prismatic cohomology

Given a smooth projective formal scheme X over $\mathcal{O}_K$ where K is a p–adic field and p is an odd prime, we describe an upper bound for ramification of the mod p representations $H^i_{\acute{e}t}(X_{\overline{K}}, \mathbb{Z}/p\mathbb{Z})$ in terms of p, i, and e, the absolute ramification index of K, without any restriction on the size of i and e. In order to achieve this, a crucial input is the recently developed prismatic cohomology in its Breuil-Kisin and A_{inf}-instances, $H^i_{\Delta}(X/\mathfrak{S})$ and $H^i_{\Delta}(X_{A_{\text{inf}}}/A_{\text{inf}})$, resp., and a series of conditions $(\text{Cr}_s)_{s\geq 0}$ that control the Galois action on the elements of the Breuil-Kisin cohomology groups inside the A_{inf}-ones.

Matilde Lalín, University of Montreal
Sums of certain arithmetic functions over $\mathbb{F}_q[T]$ and symplectic distributions

In 2018 Keating, Rodgers, Roditty-Gershon and Rudnick established relationships of the mean-square of sums of the divisor function $d_k(f)$ over short intervals and over arithmetic progressions for the function field $\mathbb{F}_q[T]$ to certain integrals over the ensemble of unitary matrices when $q \to \infty$. We study two problems: the average over all the monic polynomials of fixed degree that yield a quadratic residue when viewed modulo a fixed monic irreducible polynomial P, and the average over all the monic polynomials of fixed degree satisfying certain condition that is analogous to having an argument (in the sense of complex numbers) lying at certain specific sector of the unit circle. Both problems lead to integrals over the ensemble of symplectic matrices when $q \to \infty$. We also consider analogous questions involving convolutions of the von Mangoldt function. This is joint work with Vivian Kuperberg.

Wanlin Li, CRM Montreal
A generalization to Elkies's theorem

Elkies proved that for a fixed elliptic curve over $\mathbb{Q}$, there exists infinitely many primes at which its reductions are supersingular. In this talk, we give the first generalization to Elkies's theorem for some curves of genus > 2. We consider families of cyclic covers of the projective line ramified at 4 points whose moduli space is embedded in a Shimura curve. This is joint work in progress with Elena Mantovan, Rachel Pries, and Yunqing Tang.

Evan O'Dorney, University of Notre Dame
Reflection theorems for number rings

Scholz's celebrated 1932 reflection principle, relating the 3-torsion in the class groups of $\mathbb{Q}(\sqrt{D})$ and $\mathbb{Q}(\sqrt{-3D})$, can be viewed as an equality among the numbers of cubic fields of different discriminants. In 1997, Y. Ohno discovered (quite by accident) a beautiful reflection identity relating the number of cubic rings, equivalently binary cubic forms, of discriminants D and $-27D$, where D is not necessarily squarefree. This was proved in 1998 by Nakagawa, but the proof is rather opaque. In my talk, I will present a new and more illuminating method for proving identities of this type, based on Poisson summation on adelic cohomology (in the style of Tate's thesis). I have found extensions to quadratic forms and quartic forms and rings and to the function-field setting, where they relate threefold (possibly singular) covers of a curve.

Sudhir Pujahari, University of Warsaw
An all-purpose Erdős-Kac theorem

In this talk we will derive an Erdős-Kac type result in a very general setting and use it to recover Erdős-Kac kind theorems in many different settings. In particular, we will obtain an Erdős-Kac kind result for sums of eigenvalues of Hecke operators. This is a joint work with M. Ram Murty and V. Kumar Murty.

Anwesh Ray, University of British Columbia
Constructing Galois representations ramified at one prime

A prime p is said to be regular if p does not divide the class number of $\mathbb{Q}(\mu_p)$. Given an integer $n > 1$ and a prime p greater than or equal to $4\lfloor n/2 \rfloor + 1$, Ralph Greenberg constructed a Galois representation $\rho : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_n(\mathbb{Z}_p)$ with large image and with minimal ramification. In fact, ρ is constructed so as to be unramified at all primes $\ell \neq p$. We prove that such Galois representations (ramified at one prime) can be constructed for irregular primes as well, provided certain bounds are satisfied for the index of irregularity. The results are proved by certain suitable mod-p Galois representations with image in the diagonal torus inside $\mathrm{GL}_n(\mathbb{F}_p)$. The results are of interest from the perspective of the Inverse Galois problem.

Manami Roy, Fordham University
Rational elliptic curves with non-trivial torsion

We consider a parameterized family E_T of elliptic curves with the property that they parameterize all elliptic curves $E/\mathbb{Q}$ which contain T in their torsion subgroup, where T is one of fourteen possible non-trivial torsion subgroups allowed by Mazur's Torsion Theorem. Using these parameterized families and Tate's Algorithm, we explicitly classify the Néron types, the conductor exponents, and the local Tamagawa numbers of $E/\mathbb{Q}$ at the primes p where it has additive reduction. Consequently, we find all rational elliptic curves with a 2-torsion and 3-torsion point which have global Tamagawa number equal to 1. This is a joint work with Alexander J. Barrios.

Cecília Salgado, University of Groningen
Mordell-Weil rank jumps on families of elliptic curves

In this talk I will discuss recent progress around the variation of the Mordell-Weil rank in families of elliptic curves. The first part will be dedicated to give an introduction to the theme, by motivating, discussing the state of the art and the different techniques to tackle this question. The second part will be dedicated to presenting a recent work joint with Dan Loughran (Bath) and work in progress with Renato Dias.

Ashvin A. Swaminathan, Princeton University
2-Selmer groups, 2-class groups, and the arithmetic of binary forms

We introduce a new orbit parametrization for square roots of the class of the inverse different in rings cut out by binary forms. This parametrization has many applications to studying class groups of orders in number fields, as well as rational/integral points on varieties/stacks. As a particularly interesting example, in joint work with Bhargava and Shankar, we apply the parametrization to prove that the second moment of the size of the 2-Selmer group of elliptic curves is at most 15, confirming a conjecture of Poonen and Rains.

Cuyler Warnock, University of South Carolina
Hecke operators on modular forms with eta-multiplier

The Dedekind-eta function, $\eta(z) = q^{1/24}\prod(1-q^n)$, plays a central role in the study of modular forms. An eta-quotient of level $N \geq 1$ is $f_{N,\mathbf{v}}(z) = \prod_{\delta|N} \eta(\delta z)^{r_\delta}$, where $\mathbf{v} = \langle r_\delta \in \mathbb{Z} : \delta \mid N\rangle$. For a suitable integer $D \mid 24$ and vector $\mathbf{v}$, we see that $f_{N,\mathbf{v}}(Dz)$ is a holomorphic modular form of integer weight. In this talk, we show how, for primes $\ell \geq 5$ and for $N \in \{1,2,3,4,5,6,8,9\}$, the Hecke operators T_ℓ permute the spaces

$$f_{N,\mathbf{v}}(Dz) \cdot M_w(\Gamma_0(N),\chi) \mid V_D = \{f_{N,\mathbf{v}}(Dz)F(Dz) : F(z) \in M_w(\Gamma_0(N),\chi)\}.$$

The proof requires a careful study of how Hecke operators act on spaces of modular forms which transform with respect to the η-multiplier system. This is joint work with Matthew Boylan.

Zhining Wei, Ohio State University
Linear relations of Siegel-Poincaré Series and nonvanishing of the central value of spinor L-functions

In this paper, we will first investigate the linear relations of a one parameter family of Siegel-Poincaré series. Then we give the applications to the non-vanishing of Fourier coefficients of Siegel cusp eigenforms and the central values.

Shaoyun Yi, University of South Carolina
Siegel modular forms of level 4

In this talk, we present explicit dimension formulas of Siegel cusp forms for certain congruence subgroups of level 4. To obtain these desired dimension formulas, we will explore the connection between Siegel cusp forms of degree 2 and cuspidal automorphic representation of GSp(4). This is joint work with Manami Roy and Ralf Schmidt.
