PAlmetto **N**umber **T**heory **S**eries **XXXVI**

in memoriam of Kevin James

SCHEDULE OF ACTIVITIES

Talks will take place in Martin M1 (in the basement of Martin Hall–M), Martin M102, M103.
Refreshments are available during all breaks in Martin M1.
All invited talks are accessible via zoom using the link:
`https://clemson.zoom.us/j/92625428583?pwd=b2NWYVRRaWk0Q2FSVGxuRnY1WVQzQT09`

Saturday, October 21, 2023

8:30–8:50 [Martin M1] Coffee and other refreshments

8:50–9:00 [Martin M1] Welcome from Clemson Division Head for Mathematics, **Leo Rebholz**

9:00–9:50 [Martin M1] **Andrew Granville**, *On the mathematical works of Kevin James*. Link: `https://clemson.zoom.us/j/92625428583?pwd=b2NWYVRRaWk0Q2FSVGxuRnY1WVQzQT09`

10:00-10:50 [Martin M1] **Ken Ono**, *Counting finite field matrix points on curves and surfaces*

11:20–12:10 [Martin M1] **Olivia Beckwith**, *Imaginary quadratic fields with p-torsion-free class groups and specified split primes*

12:10-1:40 Lunch

1:40–2:30 [Martin M1] **David Penniston**, *Partitions and complex multiplication*

2:50-3:40 [Martin M1] **Carl Pomerance**, *The shifted prime divisor function*

4:00-4:20 [Martin M1] **Andreas Mono**, *Central L-values of newforms and local polynomials*

[Martin M102] **Sushmanth Jacob Akkarapakam**, *On the Ramanujan's cubic continued fraction*

[Martin M103] **Tom Wright**, *Prime gaps and Siegel zeroes - further adventures*

4:30–4:50 [Martin M1] **Rajat Gupta**, *Summation formulas attached Hecke's functional equations*

[Martin M102] **Tsz Chan**, *Powerful and powered numbers in short intervals*

5:10–6:00 [Martin M1] **Emma Lien**, *Hypergeometric Character Sums and Artin Representations*

The organizers thank NSF, NSA, and SMSS and College of Science of Clemson University for their support.

Sunday, October 22, 2023

8:30-9:00 [Martin M1] Coffee and other refreshments

9:00–9:50 [Martin M1] **Scott Ahlgren**, *Congruences for the partition function*

10:10-11:00 [Martin M1] **Matt Papanikolas**, *Traces of singular moduli for Drinfeld modules*

11:30–11:50 [Martin M1] **Rahul Kumar**, *Kronecker limit formula for real quadratic fields*

[Martin M102] **Hyuk Jun Kweon**, *Bounds on the Torsion Subgroups of Second Cohomology*

[Martin M103] **Kalani Thalagoda**, *Computing Bianchi Modular Forms*

12:00–12:20 [Martin M1] **Noah Lebowitz-Lockard**, *On the kth smallest parts of partition*

[Martin M102] **Lee Troupe**, *Erdös-Kac Theorems*

[Martin M103] **Haiyang Wang**, *On the Kodaira Types of Elliptic Curves with Potentially Good Supersingular Reduction*

12:40-1:00 [Martin M1] **Larry Rolen**, *L-functions for harmonic Maass forms*

[Martin M102] **Hannah Powell**, *Asymptotic of the Over Power Partition*

[Martin M103] **Jonah Klein**, *Bounding the j-th smallest modulus of a covering system with distinct moduli*

1:10–1:30 [Martin M1] **Brian Grove**, *Hypergeometric Moments and Hecke Trace Formulas*

[Martin M102] **Shiva Chidambaram**, *Computing the exceptional primes for torsion Galois representations of Picard curves*

1:30 **END OF CONFERENCE**

Abstracts

Scott Ahlgren, UIUC, *Congruences for the partition function*

The partition function $p(n)$ counts the number of ways to break a natural number n into parts. The arithmetic properties of this function have been the topic of intensive study since Ramanujan famously proved that the numbers $p(5n+4)$ are always divisible by 5. Much of the interest (and the difficulty) in this problem arises from the fact that values of the partition function are given by the coefficients of a modular form of negative half integral weight.
I'll describe recent results with Olivia Beckwith, Martin Raum, Patrick Allen, Shiang Tang, Nick Andersen and Robert Dicks which go a long way towards explaining exactly when such congruences can occur. The tools involve the theory of modular forms, Galois representations, and analytic number theory.

Sushmanth Jacob Akkarapakam, University of Missouri-Columbia, *On the Ramanujan's cubic continued fraction*

We study the Ramanujan's cubic continued fraction $c(\tau)$ and describe how the periodic points for prescribed function arise as values of this continued fraction. We let R_K be the ring of integers in this field and the prime ideal factorization of (3) in R_K be $(3) = \mathfrak{p}_3\mathfrak{p}_3'$. Denoting by Ω_f the ring class field over K whose conductor is f corresponding to the order R_{-d} of discriminant $-d = d_K f^2$ in K, (d_K is the discriminant of K), we show that certain values of $c(\tau)$, for τ in the imaginary quadratic field $K = \mathbb{Q}(\sqrt{-d})$ with discriminant $-d \equiv \pm 1 \pmod 3$, are periodic points of a fixed algebraic function, independent of d, and generate certain class fields $\Sigma_f \Omega_{2f}$ over K.

Olivia Beckwith, Tulane University, *Imaginary quadratic fields with p-torsion-free class groups and specified split primes*

We use Zagier's weight 3/2 Eisenstein series to prove results on the classification of Ramanujan-type congruences for Hurwitz class numbers. As an application, we show that for any odd prime p and finite set of odd primes S, there exists an imaginary quadratic field which splits at each prime in S and has class number indivisible by p. This result is in the spirit of results by Bruinier, Bhargava (when $p = 3$) and Wiles, but the methods are completely different..

Tsz Chan, Kennesaw State University, *Powerful and powered numbers in short intervals*

Powerful numbers are integers with exponents greater than 1 in their prime factorization. Barry Mazur introduced the concept of powered numbers which can be thought of as "smoothed" version of powerful numbers. In this talk, we will consider these types of numbers in short intervals. They are related to sieve method, the abc-conjecture, and Roth's theorem on arithmetic progressions.

Shiva Chidambaram, MIT, *Computing the exceptional primes for torsion Galois representations of Picard curves*

Serre's uniformity conjecture predicts that the ell-torsion Galois representation of an elliptic curve over $\mathbb{Q}$ is surjective for every prime $\ell > 37$. Given a typical genus 2 curve, Dieulefait's algorithm computes the finite set of exceptional primes ell such that the associated ell-torsion Galois representation is not surjective. The largest known exceptional prime in genus 2 is currently 31. In this talk we will discuss an algorithm to compute the exceptional primes for Picard curves $y^3 = f_4(x)$, which are genus 3 curves having an automorphism of order 3. The generic mod-ℓ Galois image for these curves is $GL(3, F_\ell)$ if $\ell \equiv 1 \pmod 3$, and $GU(3, F_\ell)$ if $\ell \equiv 2 \pmod 3$. Thus far by running our algorithm on a large database of 7-smooth Picard curves, the largest exceptional prime we find is 13, hinting that they are rarer. This is joint work with Pip Goodman.

Andrew Granville, University of Montreal, *On the mathematical works of Kevin James*

We recall some of the beautiful theorems proved by Kevin James.

Brian Grove, Louisiana State University, *Hypergeometric Moments and Hecke Trace Formulas*

An important theme throughout the work of Prof. Kevin James was his interest in problems related to the Sato-Tate conjecture. The classical Sato-Tate problem tells us the distribution of normalized trace of Frobenius values for a fixed non-CM elliptic curve over $\mathbb{Q}$ converges to a semi-circular distribution as p tends to infinity. Recently, Ono, Saad, and Saikia found the distribution is also semi-circular if we now consider all elliptic curves over $\mathbb{Q}$ in the Legendre family. Along the way, the authors define a hypergeometric moment, which can be viewed as an average of the normalized trace of Frobenius values as p tends to infinity, and establish a few cases. I will discuss how to use known Hecke trace formulas to prove additional cases of hypergeometric moments.

Rajat Gupta, University of Texas at Tyler, *Summation formulas attached Hecke's functional equations*

In this talk, I would present summation formulas attached to arithmetic functions which satisfies Hecke's functional equations. That includes, Abel-Plana summation formulas and Voronoi summation formula. As an application of these, I will particularly discuss the case when the arithmetic functions are the Fourier coefficients of cusp forms over modular group. If time permits, I would also discuss the case when arithmetic functions functions counts the sum of k-square functions.

Jonah Klein, University of South Carolina, *Bounding the j-th smallest modulus of a covering system with distinct moduli*

A covering system is a finite set of arithmetic progressions with the property that each integer belongs to at least one of them. In 2015, Hough resolved a famous problem of Erdős, showing that the smallest modulus in a covering system with distinct moduli is always smaller than 10^{16}. In 2022, Balister, Bollobás, Morris, Sahasrabudhe, and Tiba improved Hough's bound to 616000, with a method that they named the distortion method. A natural next question is to ask if the j-th smallest modulus of a covering system with distinct moduli is bounded. Using a slight modification of the distortion method and a combinatorial argument based on a theorem of Crittenden and Vanden Eynden, we show that there exists some absolute constant c such that the j-th smallest modulus of a minimal covering system with distinct moduli is $\leq \exp(\frac{cj^2}{\log(j+1)})$. This is joint work with Dimitris Koukoulopoulos and Simon Lemieux.

Rahul Kumar, Penn State, *Kronecker limit formula for real quadratic fields*

The Kronecker limit formulas are concerned with the constant term in the Laurent series expansion of certain Dirichlet series at $s = 1$. Various special functions appear in Kronecker limit formulas; such as *Herglotz function* $F(x)$, $J(x)$ and $T(x)$. These functions appeared in the works of Herglotz, Muzaffar and Willliams, and Zagier. Recently, Radchenko and Zagier extensively studied the properties of the Herglotz function, such as its special values, connection to Stark's conjecture, etc. After providing an overview of the history of this research area, we will discuss the arithmetic properties of a Herglotz-type function that appears in a Kronecker limit formula derived by Novikov. For example, we will present the two- and three-term functional equations satisfied by it along with its special values. If time permits, we will also discuss a new Kronecker limit formula for real quadratic fields. This talk is based on the joint papers with Professor YoungJu Choie.

Hyuk Jun Kweon, University of Georgia, *Bounds on the Torsion Subgroups of Second Cohomology*

Let $X \hookrightarrow \mathbb{P}^r$ be a smooth projective variety defined by homogeneous polynomials of degree $\leq d$ over an algebraically closed field k. Let $\mathbf{Pic}\, X$ be the Picard scheme of X, and $\mathbf{Pic}^0 X$ be the identity component of $\mathbf{Pic}\, X$. The Néron–Severi group scheme of X is defined by $\mathbf{NS}X = (\mathbf{Pic}\, X)/(\mathbf{Pic}^0 X)_{\mathrm{red}}$, and the Néron–Severi group of X is defined by $\mathrm{NS}\, X = (\mathbf{NS}X)(k)$. We give an explicit upper bound on the order of the finite group $(\mathrm{NS}\, X)_{\mathrm{tor}}$ and the finite group scheme $(\mathbf{NS}X)_{\mathrm{tor}}$ in terms of d and r. As a corollary, we give an upper bound on the order of the torsion subgroup of second cohomology groups of X and the finite group $\pi^1_{\mathrm{et}}(X, x_0)^{\mathrm{ab}}_{\mathrm{tor}}$. We also show that $(\mathrm{NS}\, X)_{\mathrm{tor}}$ is generated by $(\deg X - 1)(\deg X - 2)$ elements in various situations.

Noah Lebowitz-Lockard, University of Texas at Tyler, *On the kth smallest parts of partition*

For a given integer n, list all partitions of n into distinct parts. if the number of parts is odd, add the smallest part, and if the number of parts is odd, subtract it. A classic theorem of Uchimura states that this sum is equal to the number of divisors of n. We discuss the history of this result and a recent generalization to the sum over kth smallest parts for a fixed integer k. (Joint with Rajat Gupta and Joseph Vandehey).

Emma Lien, Louisiana State University, *Hypergeometric Character Sums and Artin Representations*

Given two multisets α and β of n rational numbers, one can construct two kinds of hypergeometric function; over the complex numbers and finite fields. In the complex setting, the generalized hypergeometric functions ${}_nF_{n-1}(\alpha, \beta; \lambda)$ are special functions important to both mathematics and physics. In number theory, they can be related to periods of hypergeometric varieties, $V(\alpha, \beta)_\lambda$, over number fields. In the finite field setting hypergeometric functions, $H_q(\alpha, \beta; \lambda)$, can be related to point counts of $V(\alpha, \beta)_\lambda$ over finite fields. Specifically, they correspond to traces of Galois representations; and this raises the question of when these representations are modular. The construction of these representations is very general and the methods used vary greatly depending on the "shape" of the hypergeometric data. This talk will go over some of the main results others have done and talk about the modularity of a certain 6-dimensional representation related to the weight 1 form $\eta(12\tau)^2$ and it's corresponding Artin representation.

Andreas Mono, Vanderbilt University, *Central L-values of newforms and local polynomials*

We characterize the vanishing of twisted central L-values attached to newforms of square-free level in terms of so-called local polynomials and the action of finitely many Hecke operators thereon. We provide numerical examples in weight 4 and levels $7, 15, 22$.

Ken Ono, University of Virginia, *Counting finite field matrix points on curves and surfaces*

Counting points on varieties over finite fields has a long history. Indeed, this is the theory of the local zeta function of a variety. In this talk we consider a different aspect. We consider points on varieties with coordinates in $GL_n(F_q)$, where q is fixed and $n \to +\infty$. This is the dimension aspect. In joint work with Huang and Saad, we determine these zeta functions for elliptic curves and certain families of $K3$ surfaces. Using these zeta-functions, we determine Sato-Tate distributions in dimension aspect.

Matt Papanikolas, Texas A&M University, *Traces of singular moduli for Drinfeld modules*

In the 1990's and 2000's, Zagier proved remarkable results on traces of singular moduli that showed they satisfy various simple identities on average and that ultimately they constitute the Fourier coefficients of a particular half-integral weight modular form. In the present talk we will consider ways to phrase these same questions over the rational function field in one variable over a finite field. In this setting singular moduli are defined as values of the Drinfeld modular j-invariant on Heegner points in the Drinfeld upper half-space. Through work of Bae, Hsia, Wang, J. Yu, and J.-K. Yu, it is possible to define class polynomials over the rational function field whose roots are singular moduli but which also satisfy explicit connections with modular polynomials. Building on these constructions we devise average results for traces of singular moduli and recover formulas for Hurwitz class numbers that align with Zagier's results. Joint with A. El-Guindy, R. Masri, and G. Zeng.

David Penniston, University of Wisconsin Oshkosh, *Partitions and complex multiplication*

The Ramanujan congruences provide beautiful results on the arithmetic of the unrestricted partition function, while Euler's pentagonal number theorem furnishes a complete description of the parity of the number of partitions into distinct parts. In this talk we will consider the arithmetic behavior of k-regular partitions, which are partitions in which no part is divisible by k. We give particular attention to cases where modular forms with complex multiplication can be used to characterize this behavior..

Carl Pomerance, Dartmouth College, *The shifted prime divisor function*

I consider the function which counts the number of divisors of n of the form $p-1$, where p is prime. This function has been studied by Prachar, Erdos and Prachar, Murty and Murty, and very recently, Ding. At first glance it seems completely analogous to the number of divisors that are primes, since on average they are both about loglog n. However, looking closer, they are quite different, a fact that has proven useful in primality testing and the distribution of Carmichael numbers. This is joint work with Kai (Steve) Fan.

Hannah Powell, University of North Carolina Charlotte, *Asymptotic of the Over Power Partition*

TBA.

Larry Rolen, Vanderbilt Universtity, *L-functions for harmonic Maass forms*

The theory of harmonic Maass forms and mock modular forms has seen an explosion of activity in the past 20 years, with applications to physics, partitions, enumerative geometry, and many other topics. Along the way, much has been developed in the theory of harmonic Maass forms. However, until recently, harmonic Maass form theory lacked analogues of key structures that exist for classical holomorphic modular forms and Maass waveforms, such as the theory of L-functions. Recent work with Diamantis, Lee and Raji will be described which gives the first general such theory. In particular, I will sketch Weil-type Converse Theorems and a Voronoi-type summation formula in these settings. I will also describe connections with the construction of differential operators on these spaces and a more thorough explanation of a previous formula for a central L-value of the L-invariant, which had been discovered heuristically by Zagier and proven in that case by Bruinier, Funke, and Imamoglu.

Kalani Thalagoda, Tulane University, *Computing Bianchi Modular Forms*

In this talk, I will go over an algorithm for computing Bianchi modular forms over the field $\mathbb{Q}(\sqrt{-17})$.

Lee Troupe, Mercer University, *Erdős-Kac Theorems*

Let $\omega(n)$ denote the number of distinct prime factors of a natural number n. The celebrated Erdős-Kac theorem says that, as n ranges over the natural numbers, the quantity $\omega(n)$ satisfies a normal distribution with mean and variance $\log \log n$. Variations on and generalizations of the Erdős-Kac theorem abound. For instance, what if $\omega(n)$ is replaced by $\omega(f(n))$, where $f(n)$ is some arithmetic function? We'll discuss answers to that question for various choices of the function $f(n)$, including some that lack certain convenient properties (e.g. additivity).
Includes joint work with Paul Pollack, University of Georgia; and Greg Martin, University of British Columbia.

Haiyang Wang, University of Georgia, *On the Kodaira Types of Elliptic Curves with Potentially Good Supersingular Reduction*

Let O_K be a Henselian discrete valuation domain with field of fractions K. Assume that O_K has algebraically closed residue field k. Let E/K be an elliptic curve with additive reduction. The semi-stable reduction theorem asserts that there exists a minimal extension L/K such that the base change E_L/L has semi-stable reduction.
It is natural to wonder whether specific properties of the semi-stable reduction and of the extension L/K impose restrictions on what types of Kodaira type the special fiber of E/K may have. In this talk we will discuss the restrictions imposed on the reduction type when the extension L/K is wildly ramified of degree 2, and the curve E/K has potentially good supersingular reduction. We will also discuss the possible reduction types of two isogenous elliptic curves with these properties.

Tom Wright, Wofford College, *Prime gaps and Siegel zeroes - further adventures*

In this talk, we discuss some more recent results in the study of prime gaps under the assumption of Siegel zeroes.