Homework 1, due: 01/27

MATH 9830, Spring 2015 Timo Heister, heister@clemson.edu

1. Write down the pseudo code to compute the product of the transpose sparse matrix in CSR format with a vector:
$$y = A' \cdot x$$
Do not use the naive way by searching for all non-zero entries in column i.

2. Take the "01_sparse_mat" source code from the class repo and implement your pseudo code in 1).

3. Create the directed graph for the 1d finite difference boundary values problem:
$$\begin{pmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & \ddots & \ddots & \ddots \\ & & -1 & 2 \end{pmatrix}$$
for $n = 5$ and proceed to eliminate the second and fourth column (above and below the diagonal). Draw the final graph.

4. Let A be a symmetric matrix, $LL^T = A$ the Cholesky decomposition, and $G(A)$ the undirected graph of A. Show: $(i, j) \in G(L + L^T)$ if and only if there exists a path from i to j in $G(A)$ with all nodes (except i and j) have number smaller than $\min(i, j)$.

5. Assume you have a regular finite element mesh based on quads in dimension $d = 2$ or $d = 3$ (so each vertex has 4 cells in 2d and 8 in 3d). Give a best-case estimate for the half-bandwidth p of the system matrix for a linear finite element space. Assuming that LU decomposition takes $O(p^2 n)$ time and you can solve a 2d problem with $n = 10000$ in 1 second, discuss the amount of time necessary for $n = 1$ million and $n = 100$ million in 2d and 3d.

   Bonus: what about a quadratic element?