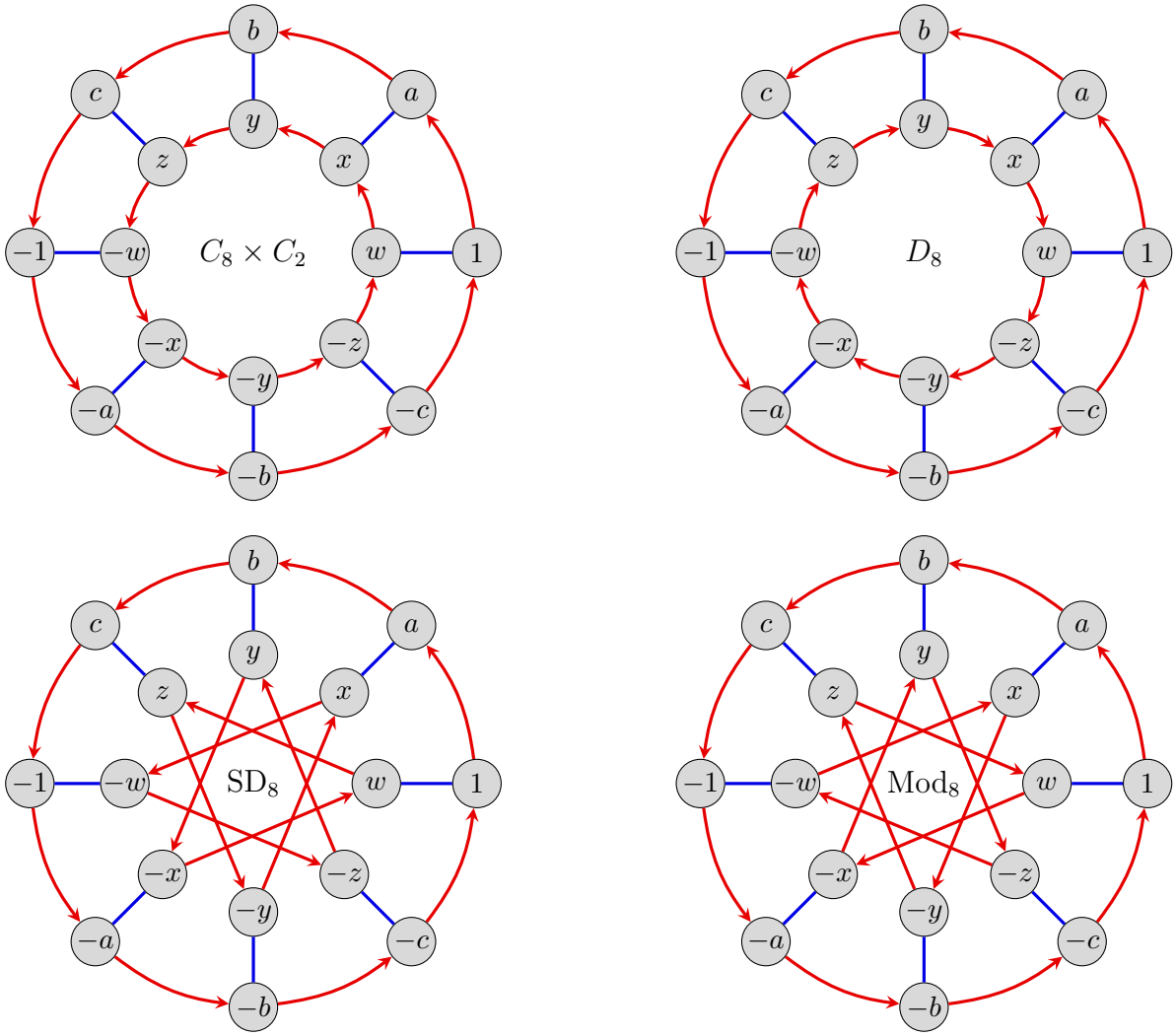
1. Cayley diagrams for the four semidirect products of C_8 with C_2 are shown below, with a different labeling scheme on their nodes.



For all four of these groups, identifying each element with its "negative" yields a "quotient group" of order 8, like what we did with the dicyclic group $\mathrm{Dic}_8 = Q_{16}$ in HW 2. Construct a Cayley table and Cayley diagram for each of these quotient groups, using the elements

$$\pm 1, \quad \pm a, \quad \pm b, \quad \pm c, \quad \pm w, \quad \pm x, \quad \pm y, \quad \pm z,$$

and determine to which familiar group each is isomorphic.

2. In this problem, we will see what happens if we tweak our standard representations and try to create a dicyclic group with an odd root of unity, or a semidihedral or modular group with $n \neq 2^m$. Determine what the following groups are actually isomorphic to.

(a) $\mathrm{Dic}_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & \bar{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle$

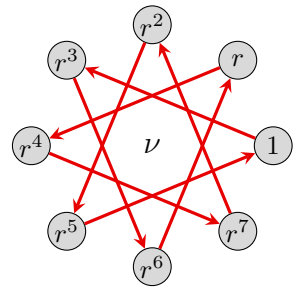
(b) $\mathrm{SD}_3 := \left\langle \begin{bmatrix} \zeta_3 & 0 \\ 0 & -\bar{\zeta}_3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle$

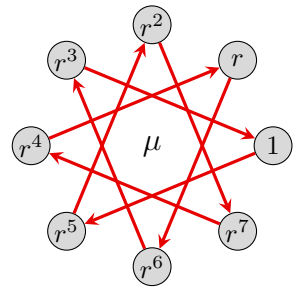
(c) $\mathrm{SD}_6 := \left\langle \begin{bmatrix} \zeta_6 & 0 \\ 0 & -\bar{\zeta}_6 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle$

(d) $\mathrm{Mod}_6 := \left\langle \begin{bmatrix} \zeta_6 & 0 \\ 0 & -\zeta_6 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle.$

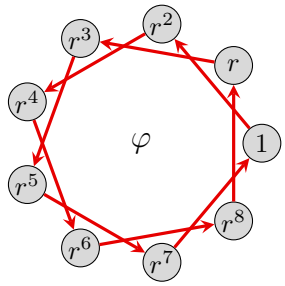
3. The automorphism group of C_n is isomorphic to $U(n)$, the multiplicative group of integers modulo n, from HW 2, #2. For each of the following, construct a Cayley diagram of $\mathrm{Aut}(C_n)$ with the nodes labeled by re-wirings, and a Cayley table for this group.

   (a) $\mathrm{Aut}(C_8) = \langle \nu, \mu \rangle$, defined by

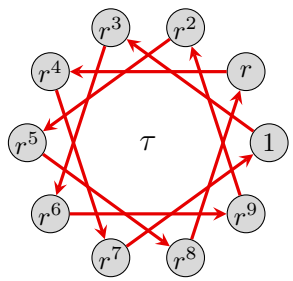




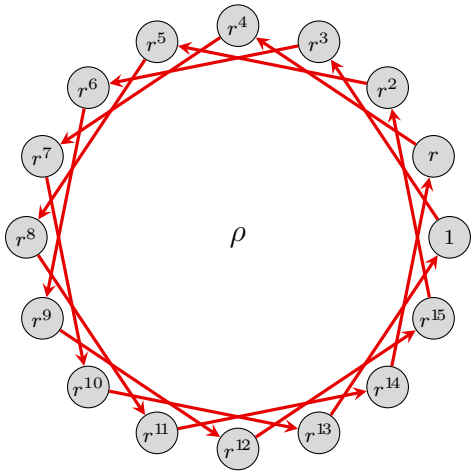
   (b) $\mathrm{Aut}(C_9) = \langle \varphi \rangle$, defined by

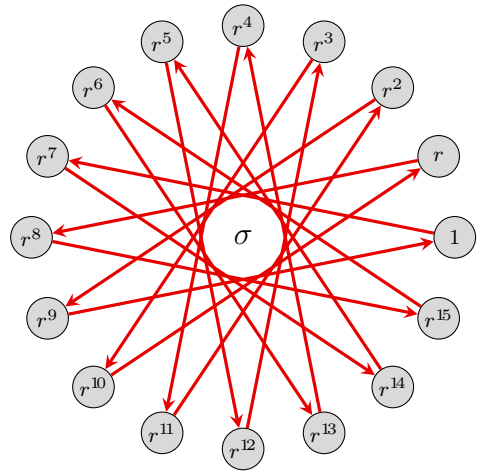


   (c) $\mathrm{Aut}(C_{10}) = \langle \tau \rangle$, defined by



   (d) $\mathrm{Aut}(C_{16}) = \langle \rho, \sigma \rangle$, defined by





4. Using your answer to Part (b) of the previous problem, construct a nonabelian semidirect product of $C_9 = \langle r \rangle$ with $C_3 = \langle s \rangle$. Make sure to explicitly define your labeling map $\theta \colon C_3 \to \mathrm{Aut}(C_9)$. Include a Cayley diagram of C_3 with the nodes labeled by $\theta(s^j)$, and a Cayley diagram of $C_9 \rtimes_\theta C_3$, with the nodes labeled by $r^i s^j$.