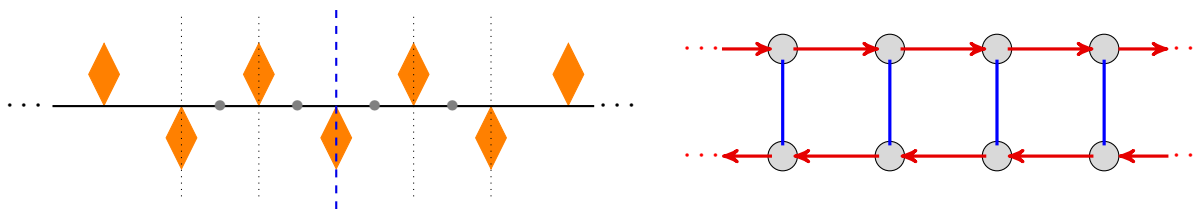


Read Chapters 3–4 of *Visual Group Theory*, or Chapters 5.1–5.3 of *IBL Abstract Algebra*. Then write up solutions to the following exercises.

1. Consider the frieze pattern shown below at left.

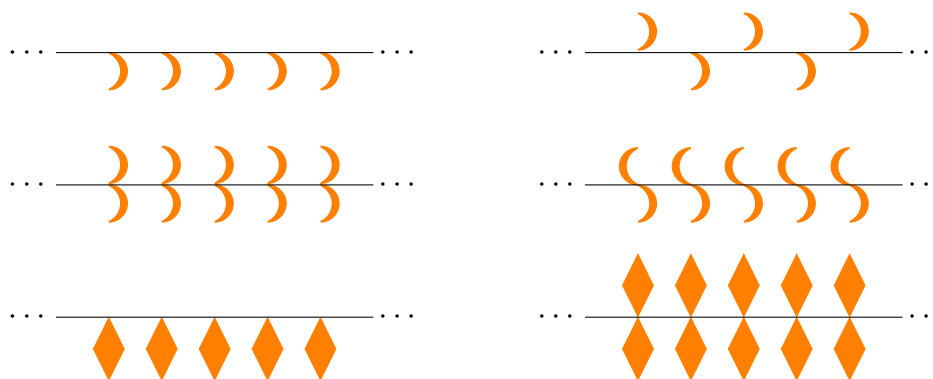


Let g be a glide-reflection to the right, and h a horizontal flip about the dashed blue line. These actions (symmetries) generate a *frieze group*, and the Cayley diagram is shown on the right. Express each the following symmetries in terms of g and h :

- (a) The *reflection* about each dotted line.
- (b) The 180° *rotation* around each gray dot.

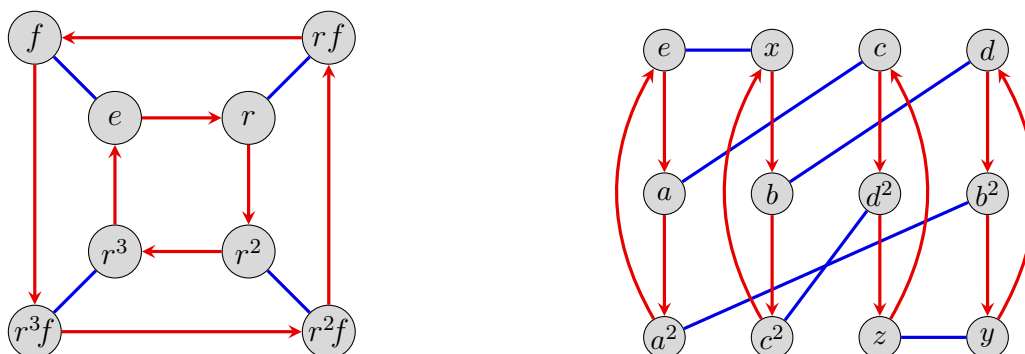
If possible (it may not be), write your answer as $g^k h g^{-k}$ for some $k \in \mathbb{Z}$.

2. There are 7 frieze groups. In addition to the one in the previous problem, the other 6 are symmetry groups of the friezes shown below.



For each of these friezes, draw a Cayley diagram for its frieze group using a minimal generating set. Make it clear what the generators are and write out a group presentation.

3. Shown below are the Cayley graphs of two groups: D_4 is on the left, and the right is called an “alternating group,” denoted A_4 .



- (a) Create a multiplication table for each group. For consistency, order the elements in D_4 by $(e, r, r^2, r^3, f, rf, r^2f, r^3f)$ and by $(e, x, y, z, a, a^2, b, b^2, c, c^2, d, d^2)$ in A_4 .
- (b) Find the inverse of each element of each group.
- (c) Write out a group presentation for each group using the generators shown in the Cayley graph.
4. In each of the following multiplication tables, let e denote the identity element. Complete each table so it depicts a group. There may be more than one way to complete a table, in which case you need to give all possibilities. Draw a Cayley diagram for each.

	e	a
e		
a		

	e	a	b
e			
a			
b			

	e	a	b	c
e				
a		e		
b				
c				

5. Show that an element cannot appear twice in the same column of a multiplication table.
6. Using our “unofficial” definition of a group, show that every group has a unique identity action e , satisfying $ge = g = eg$ for every action g in G . [*Hint*: You need to show both existence and uniqueness. For the latter, assume that e and f are both identity actions. Can you show that $e = f$?]
7. Let \mathbb{Z} , \mathbb{Q} , and \mathbb{R} denote the set of integers, rational numbers, and real numbers, respectively. Let \mathbb{Z}^+ , \mathbb{Q}^+ , and \mathbb{R}^+ denote the positive integers, rationals, and reals. Let \mathbb{Z}^* , \mathbb{Q}^* , and \mathbb{R}^* denote the nonzero integers, rationals, and reals.
- (a) Which of the above sets are groups under addition? For each one that is a group, give a minimal generating set if there is one. For each one that is not, give an explicit reason for why it fails.
- (b) Which of the above sets are groups under multiplication? For each one that is a group, give a minimal generating set if there is one. For each one that is not, give an explicit reason for why it fails.
- (c) Let $n\mathbb{Z}$ denote the set of all integers that are multiples of n . For what $n \in \mathbb{N}$ is the set $n\mathbb{Z}$ a group under addition? Give a minimal generating set for each one that is a group.